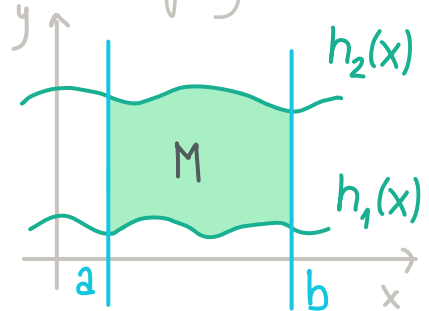


DVOJNÉ INTEGRÁLY

- FUBINIOVA VĚTA - dvojný integrál na dvojité

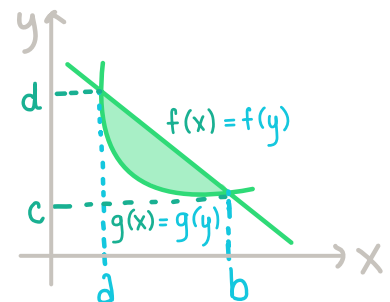
$$\int_M f(x,y) dA = \int_a^b \int_{h_1(x)}^{h_2(x)} f(x,y) dy dx$$

a, b, \dots čísla \rightarrow konst. meze nakonec



- OPAČNÉ POŘADÍ INTEGRACE

$$\int_a^b \int_{g(x)}^{f(x)} \dots dy dx \quad \begin{matrix} \circ x \rightarrow \\ \uparrow y \circ \end{matrix} \quad \int_c^d \int_{g(y)}^{f(y)} \dots dx dy$$



- SUBSTITUCE

$$\iint f(x,y) dx dy \rightarrow \iint f(u,v) |J| du dv$$

$J \neq 0$ Jakobián (determinant)

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \quad J(x,y) = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$J(u,v) = \frac{1}{J(x,y)}$$

INVERZNÍ ZOBRAZENÍ

- POLÁRNÍ SOUŘADNICE

$$\iint f(x,y) dy dx \rightarrow \iint f(r,\varphi) \cdot |J| dr d\varphi$$

\rightarrow lepší oblast integrace: z kružnice na obdélník

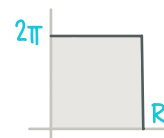
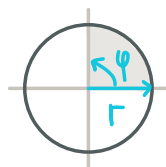
$$x = r \cdot \cos \varphi \quad x = a \cdot r \cdot \cos \varphi$$

$$y = r \cdot \sin \varphi \quad y = b \cdot r \cdot \sin \varphi$$

$$J = r \quad J = a \cdot b \cdot r$$

KRUŽNICE

ELIPSA



$$r > 0$$

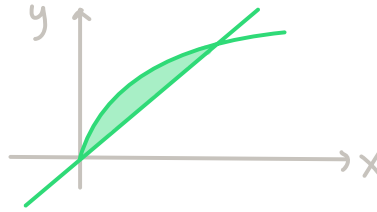
poloměr (vždy kladný)



APLIKACE DVOJNÉHO INTEGRÁLU

- OBSAH OBRAZCE

$$S = \int_M 1 \, dA$$

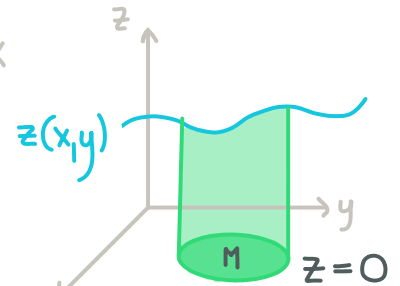


(jako MAZ)

- OBJEM

$$V = \int_M z \, dA$$

$$z = f(x, y)$$



- HMOTNOST

$$m = \int_M h \, dA$$

$$h = f(x, y)$$

↳ plošná hustota

- STATICKÝ MOMENT

- k ose x: $S_x = \int_M y \cdot h \, dA$

- k ose y: $S_y = \int_M x \cdot h \, dA$

- TĚŽIŠTĚ

$$x_T = \frac{S_y}{m}$$

$$y_T = \frac{S_x}{m}$$

} T [x_T, y_T]

- MOMENT SETRVAČNOSTI

- k ose x: $I_x = \int_M y^2 \cdot h \, dA$

- k ose y: $I_y = \int_M x^2 \cdot h \, dA$

- k počátku: $I_z = I_x + I_y = \int_M (x^2 + y^2) \cdot h \, dA$

- OBSAH PLOCHY

$$P = \int_M \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \, dA$$

